# RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

**FIRST YEAR [2017-20]**
**B.A./B.Sc. FIRST SEMESTER (July – December) 2017**
**Mid-Semester Examination, September 2017**

Date : 12/09/2017 **MATHEMATICS (Honours)**
Time : 11 am – 1 pm Paper : I Full Marks : 50

**[Use a separate Answer Book for each group]**

## Group – A

1. Answer **any three** questions : [3×4]

   a) If $x, y \in \mathbb{R}$ with $y > 0$ then prove that $\exists\, n \in \mathbb{N}$ such that $ny > x$.

   b) Prove that the union of a finite number of open sets in $\mathbb{R}$ is open.

   c) For any set A in $\mathbb{R}$ prove that $(A')' \subset A'$ i.e., the derived set of any set in $\mathbb{R}$ is closed.

   d) Let G be an open set in $\mathbb{R}$. Show that the intersection of any two open intervals in G is either an empty set or contains more than one point.

   e) Prove that the sequence $\{u_n\}_n$ defined by $u_1 = \sqrt{2}$ and $u_{n+1} = \sqrt{2u_n}$ $\forall n \in \mathbb{N}$ converges to 2.

2. If $ax^2 + 2hxy + by^2 = 0$ are two sides of a triangle and $(\alpha, \beta)$ is its orthocentre, prove that the equation of the third side is $(a+b)(\alpha x + \beta y) = a\beta^2 - 2h\alpha\beta + b\alpha^2$. [6]

   **OR,**

   Find the equation of the tangent to the conic $\dfrac{\ell}{r} = 1 - e\cos\theta$ at the point with vectorial angle $\alpha$. [6]

3. a) Show by vector method that the medians of a triangle are concurrent. [3]

   b) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors then show that the four vectors $\vec{\alpha} = 6\vec{a} - 4\vec{b} + 10\vec{c}$, $\vec{\beta} = -5\vec{a} + 3\vec{b} - 10\vec{c}$, $\vec{\gamma} = 4\vec{a} - 6\vec{b} - 10\vec{c}$ and $\vec{\delta} = 2\vec{b} + 10\vec{c}$ are coplanar. [4]

   **OR,**

4. a) Show by vector method that an angle inscribed in a semi-circle is a right angle. [3]

   b) Show by vector method that $\sin(A - B) = \sin A \cos B - \cos A \sin B$. [4]

## Group – B

(*Answer **any four** of the following*) [4×3]

5. $\rho$ is defined on $\mathbb{Z}$ (set of all integers) by $a\rho b$ if and only if $a^2 - b^2$ is a multiple of 7. Is $\rho$ an equivalence relation on $\mathbb{Z}$?

6. "Every relation must either be symmetric or antisymmetric". Is the assertion right? Justify your answer.

7. a) $f = \{(a,b) \in \mathbb{R} \times \mathbb{R} \mid b = \sqrt{a}\}$. Check whether f is a mapping or not.

   b) Give an example of a map from $\mathbb{N}$ to $\mathbb{N}$ which is surjective but not injective.

8. Give example of two maps f and g which are not bijective but $g \circ f$ is bijective.

9. Prove that $\mathbb{Z}_7$ forms a group with respect to the operation "addition modulo 7".

10. Let $(G, *)$ be a group and $a, b \in G$. Suppose $a^2 = e$ and $a * b^4 * a = b^7$ where e is the identity in G. Prove that $b^{33} = e$.

## Group – C

11. Answer **any two** questions : [2×4]

a) Show that all circles of radius r are represented by the differential equation $(1+y_1^2)^{3/2} = ry_2$ where $y_1 = \frac{dy}{dx}$ and $y_2 = \frac{d^2y}{dx^2}$.

b) If the differential equation $Mdx + Ndy = 0$ has one and only one solution, then prove that there exists an infinite number of integrating factors, where M and N are any functions of x and y only.

c) Reduce the differential equation $y = 2px - yp^2$ to Clairaut's form by the substitution x = X, $y^2$ = Y and then obtain its complete primitive and singular solution, if any, where $p = \frac{dy}{dx}$.

12. Answer **any one** question : [1×5]

a) Solve the differential equation $(8p^3 - 27)x = 12p^2y$ and investigate whether a singular solution exists.

b) Using the appropriate rule, find an integrating factor of the differential equation $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$ and hence solve it.

——— × ———